191. **Proposed by J. EDWARD SANDERS, Reinersville, Ohio.**

Two random lines cut a given circle. What is the chance that they intersect within the circle?

Solution by HENRY HEATON, Belfield, N. D., and the PROPOSER.

Let x=the distance of one of the lines from the center of the circle, and θ=the angle between the lines. The length of the part of the first line lying within the circle is $2\sqrt{(a^2-x^2)}$. For given values of x and θ the chance of intersection is

$$\frac{2\sin\theta\sqrt{(a^2-x^2)}}{2a}=\frac{\sin\theta}{a}\sqrt{(a^2-x^2)},$$

and the required chance is

$$P=\int_0^a\int_0^{\frac{1}{2}\pi}\frac{\sin\theta}{a}\sqrt{(a^2-x^2)}\,dx\,d\theta\div\int_0^a\int_0^{\frac{1}{2}\pi}dx\,d\theta=\frac{2}{a\pi}\int_0^a\sqrt{(a^2-x^2)}\,dx=\tfrac{1}{2}.$$

Also solved by G. B. M. Zerr, who gets ⅛ for the result. His solution will be published in the next issue of the MONTHLY.

PROBLEMS FOR SOLUTION.

ALGEBRA.

293. **Proposed by C. E. WHITE, Vanderbilt University, Nashville, Tenn.**

Prove by mathematical induction that $\frac{(x-a)^{m-1}}{(m-1)!}f^{m-1}(a)+\frac{(x-a)^{m-2}}{(m-2)!}$ $+\ldots.\frac{(x-a)^2}{2!}f''(a)+(x-a)f'(a)+f(a)$ will be the remainder when $f(x)$ is divided by $(x-a)^m$.

294. **Proposed by O. L. CALLECOT, Gettysburg, S. Dak.**

Find the limit of $\sum_{n=1}^{n=\infty}\frac{2(n^2+3n+3)}{n(n+1)(n+2)(n+3)}$.

GEOMETRY.

326. **Proposed by L. E. NEWCOMB, Los Gatos, Calif.**

The circle C of radius pR encloses the circles A_1B_1 of radii R and $(p-1)R$, respectively; the circle B_1 is tangent to $A_1B_1C_1$; the circle B_2 is tangent to AB_1C; the circle B_3 to AB_2C, ..., B_n to $AB_{n-1}C$. Find the radius of the circle B_n.

327. Proposed by J. C. CORBIN, Pine Bluff, Ark.

In triangle ABC, the triangle DEF is formed by joining the feet of the medians and four parallelograms are also formed, viz., $AEDF$, $BFED$, and $CEFD$. Let a, b, c, d, e, f represent the three medians of ABC, and the three sides of DEF. Then the sum of the squares of the six diagonals equals the sum of the squares of the twelve sides of the parallelograms, which are equal in sets of four. That is, $a^2+b^2+c^2+d^2+e^2+f^2=4(d^2+e^2+f^2)$, or $a^2+b^2+c^2=3(d^2+e^2+f^2)=3/4(AB^2+BC^2+CA^2)$.

328. Proposed by CHARLES GILPIN, JR., Philadelphia, Pa.

A sphere with the radius R is divided into two segments by a plane passed through it half way between the center and circumference. The smaller segment is divided into two parts by a plane passed through it at right angles to the base and cutting it half way between its center and circumference. Required the contents of the two parts of the segment.

CALCULUS.

250. Proposed by V. M. SPUNAR, East Pittsburg, Pa.

Differentiate $(\log^n x)$.

251. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Find in terms of x the functions c_1x and c_2x defined, respectively, by the relations

(a) $x\log(c_1x)=c_1x\log x$,

(b) $x\log x=c_2x\log(c_2x)$.

MECHANICS.

210. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

A rigid triangle is formed of three weightless, smoothly jointed, rigid rods BC, CA, AB. At their mid points D, E, F, respectively, are small, smooth rings, through which passes an endless, stretched, elastic string, forming the triangle DEF. Find by graphical construction the reaction at the joints.

211. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

A smooth elliptic wire, axis vertical, has a small ring sliding on it, connected by elastic strings with each focus. Either string is just unstretched when the ring is nearest the corresponding focus. The modulus of elasticity is W/n, where W oz. is the weight of the ring. Find the distance of the ring from the upper focus in the different positions of equilibrium, and in each case discuss the nature of the equilibrium.

MISCELLANEOUS.

175. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

If x and z are connected by the relation $z=zf(x)+x\phi(z)$, find the value of $f(z)$ in the form of a power series in x with constant coefficients. In particular, give such a value of z when $z=z\sin x+x\cos z$.